UNIVERSITY OF ILLINOIS BULLETIN

ISSUED WEEKLY

Vol. XXVI August 6, 1929 No. 49

[Entered as second-class matter December 11, 1912, at the post office at Urbana, Illinois, under the Act of August 24, 1912. Acceptance for mailing at the special rate of postage provided for in section 1103, Act of October 3, 1917, authorized July 31, 1918.]

THE PLASTER-MODEL METHOD OF DETERMINING STRESSES APPLIED TO CURVED BEAMS

BY

FRED B. SEELY
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BULLETIN NO. 195

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BY

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ENGINEERING EXPERIMENT STATION

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THE PLASTER-MODEL METHOD OF DETERMINING STRESSES APPLIED TO CURVED BEAMS

I. Introduction

1. *The Brittle-material Method.*—The mathematical method of determining stresses in structural and machine members due to external loads may be, in many cases, inadequate or impracticable due to objectionable complications. Further, even though a mathematical analysis may yield values of the stresses at specified points in a member, the significance of these stresses in causing damage to the member is frequently not revealed by such an analysis, and therefore may be unreliable if used as the only method of determining the significant stresses in a member.

As a result of these conditions there has been developed a number of mechanical or experimental methods of investigating stresses in members, one of which is the so-called brittle-material method. This method may be explained very briefly as follows: From a "brittle" material, that has a nearly straight stress-strain diagram up to the ultimate strength of the material, a model is made of the member of more or less irregular shape in which the stress is to be found; from the same material is also made a specimen of simple shape in which the stress due to a given load can be computed satisfactorily. The specimen of simple shape and the model of complex shape are then tested to destruction. The test of the simple shape gives approximately the ultimate strength of the material and the test of the model gives, with a fair degree of accuracy, the load which produced this ultimate stress in the most stressed fiber of the model; from these values the relation between load and maximum stress is obtained.

For example, if the breaking load for a straight beam made of brittle material and having an abrupt change of section is one-half the breaking load for a similar beam without the abrupt change in section, then the maximum stress caused by a given load in the beam having the abrupt change in section is twice that in the more simply shaped beam when subjected to the same load, and this latter stress can be computed from the flexure formula.

Likewise, if the bending moment that causes rupture of a curved beam made of a brittle material is two-thirds of the bending moment that causes rupture of a straight beam of the same material and cross-section, then the maximum stress in the curved beam when subjected

to a given bending moment is one and one-half times the stress in the straight beam when subjected to the same bending moment.

With certain limitations, to be discussed later, the same relation will hold between load and maximum stress in a member made of elastic material as is found by the brittle-material method, provided that the stress does not exceed the proportional limit of the material.

The ideal material for use in the brittle-material method of determining stresses is one in which the compressive and shearing strengths are high compared to the tensile strength, and in which the stress-strain diagram is straight until rupture occurs. For such a material the proportional limit is the ultimate strength, and the material would be expected to fail whenever the tensile stress at any point in a member exceeded the proportional limit of the material, and the plane of rupture would be perpendicular to the direction of the maximum tensile stress.

Although such an ideal material is not available, "pottery plaster" meets the requirements fairly well, and it was used in the tests herein reported. Tests by W. A. Slater* and by R. E. Peterson* show that the stress-strain diagram for gypsum and pottery plaster, if properly cured, is practically a straight line up to the point of rupture, and other tests show that the compressive strength is several times the tensile strength.

2. *Purpose of Investigation.*—There were two main purposes in making the tests. The first was to determine whether or not the brittle-material method, using pottery plaster as the material, could be made to yield consistent and reliable results, particularly with members of rather irregular shapes requiring complicated molds for casting the members. For this purpose curved beams in the form of C frames (see Fig. 2) were used for the reasons that such frames as commonly made are rather complicated castings, or forgings, and that there is available a satisfactory mathematical solution for the longitudinal stress in curved beams of various sections with which to compare the test results of the plaster models.

If the results of the tests of the plaster-model method were shown to be reliable for these curved beams, then this method might be used with confidence to obtain at least approximate values of the significant stresses in other irregular members for which no satisfactory mathematical analysis is available. Also it could be used with reasonable confidence in accomplishing the second purpose of the tests.

*See Bibliography.

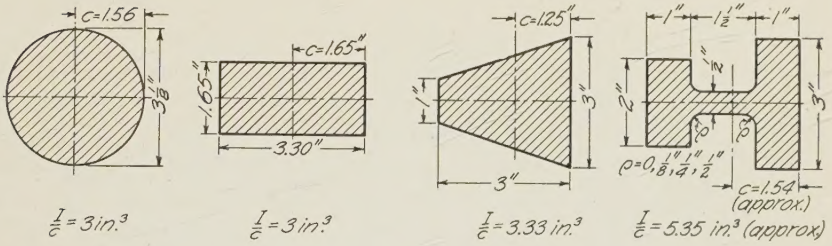


FIG. 1. APPROXIMATE DIMENSIONS OF CROSS-SECTIONS

The second purpose was to investigate the radial tensile stress in a curved beam. Certain sections of not unusual proportions for C frames may have calculated values of the maximum radial tensile stresses greater than the maximum circumferential tensile stresses. Such sections will in general have abrupt changes in width of the sections, such as T sections, I sections, and H sections. Further, if a plaster model of such a shape fails at the junction of flange and web, when subjected to a load causing a calculated value of the radial stress less than the ultimate strength of the material, then the value of the stress concentration due to the sharp corner or fillet at the junction of flange and web may be obtained. For this purpose a curved beam having an H section (see Fig. 1) was used, so designed that the maximum value of the radial stress as given by mathematical analysis occurred at the abrupt change in width of the section. Four different fillets were used: one with approximately zero radius (a sharp corner), one with a ⅛-in. radius, one with a ¼-in. radius, and one with a ½-in. radius.

3. *Acknowledgment.*—The greater portion of the tests herein reported were made by Mr. R. V. James as a graduate student at the University of Illinois in satisfying the requirement for thesis work for the degree of Master of Science in Theoretical and Applied Mechanics. Very helpful information on the properties of pottery plaster was contributed by Mr. T. N. McVay, Associate in Ceramic Engineering, and valuable assistance in mold making was given by Mr. E. G. Bourne, Laboratory Demonstrator in Ceramic Engineering. Acknowledgment is also made to Messrs. W. M. Lansford, J. C. Ducommun, E. C. Clark, N. H. Barnard, and H. J. Stoever, graduate students, for computations giving the distribution of radial stress in various cross-sections of curved beams with different degrees of curvature.

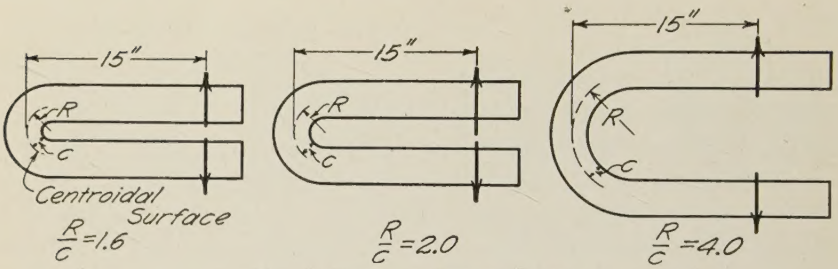


FIG. 2. FORM OF CURVED BEAMS

The tests herein reported were made as part of the work of the Engineering Experiment Station of which DEAN M. S. KETCHUM is the director and of the Department of Theoretical and Applied Mechanics of which PROF. M. L. ENGER is the head.

II. MATERIAL AND SPECIMENS USED

4. *General Properties of the Material.*—The material used for the plaster models was a high grade pottery plaster. This plaster has the same chemical composition as has plaster of paris, except for an agent which retards the setting of the pottery plaster, thereby giving ample time for working and placing.

As in many other cements the strength and quality of pottery plaster may be greatly affected by the method of handling. Within limits, the density and strength increase as the amount of mixing water decreases, and the time of setting decreases materially as the amount of mixing water decreases. The compressive strengths of pottery plaster for different water plaster ratios are about as follows:

61.5 lb. water per 100 lb. plaster, 2400 lb. per sq. in.
75 lb. water per 100 lb. plaster, 1640 lb. per sq. in.
94 lb. water per 100 lb. plaster, 850 lb. per sq. in.

The time of blending, that is, the time elapsing between putting the plaster in the water and the beginning of the stirring operation, is also an important factor affecting the strength and quality of the specimen or casting. The strength was found to increase with the blending time up to fourteen minutes when 70 per cent of water was used for mixing, and to begin to decrease rapidly as the blending time exceeded sixteen minutes.

It is desirable that the stirring be continuous and of such a character that air bubbles will be carried up from the bottom and liberated without entrained air being carried into the mixture. Violent stirring is neither necessary nor desirable.

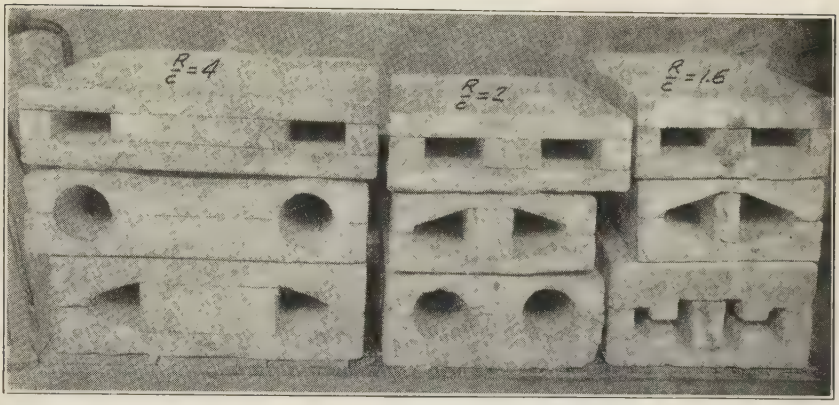


FIG. 3. MOLDS ASSEMBLED

The strength of the plaster increases rapidly with the length of time of curing in saturated air up to two days, after that length of time in the moist room further increase in strength is scarcely perceptible.

Set plaster is very easily burned (dehydrated) in drying, especially after it has become rather dry. The maximum drying temperature cannot be above 120 deg. F. without danger of burning. Burned plaster is chalky and very weak. Repeated wetting of the plaster after it has set, especially after once being dried, causes the strength to decrease markedly.

5. *Method of Mixing, Pouring, and Curing.*—In the manufacture of the specimens care was taken to secure uniformity in quality and strength of the plaster. The mixture used was 70 lb. of water per 100 lb. of plaster. After the two ingredients were weighed the plaster was poured carefully in the water and the mixture was allowed to stand for 10 minutes before being disturbed. After this 10-minute period, stirring was begun by using the cupped hand so as to stir air bubbles toward the top. The hand was kept below the surface of the liquid. The stirring was continued steadily for 5 minutes; the mixture was then ready to pour into the molds (see Fig. 3) which had been thoroughly sized with soap solution to prevent sticking of the plaster to the mold. The two parts of the mold were clamped together and placed on end, allowing the plaster to be poured in one leg of the mold and to rise in the other leg. The pouring was done as steadily as possible to prevent trapping of air. Immediately after pouring, the

material was agitated with a stick or a wire thrust to the bottom of the mold and also by tipping and jarring the mold. This procedure brought most of the larger entrained air bubbles to the surface. This agitation was stopped at the first signs of stiffening and the material allowed to set. As the material sets there is a slight expansion which causes all parts of the mold to fill nicely. After about 15 minutes the material begins to give off heat and feels quite warm to the hand. At this time the casting is sweating and a film of water is formed between the casting and the soaped surface of the mold. When in this condition the molds are most easily removed. After the specimens were taken from the molds they were marked and placed in the moist room for curing.

All specimens were left in the moist room at least two days. They were then brought out and placed over a sand drier in such a manner that the air could have free circulation around them. They were dried in this manner at a temperature slightly above room temperature for a minimum period of four days; this procedure, with few exceptions, produced hard specimens with a distinctly metallic ring when struck, and apparently free from shrinkage stresses.

6. *Form and Size of Specimens.*—Specimens with four different cross-sections were used, namely, circular, rectangular, trapezoidal, and H sections (see Fig. 1). The specimens were made in the form of an open C frame (see Fig. 2) consisting of two straight portions connected by a curved portion at one end. Three different curvatures for the curved portion were used which may be expressed by the ratios of the radii of curvature R of the centroidal surfaces to the distances c from the centroidal axes of the sections to the extreme fibers in tension. For the sharpest curvature this ratio $\frac{R}{c}$ was 1.6, which represents about the sharpest curvature likely to be encountered, and probably about the sharpest curvature for which the Winkler-Bach formula will yield reliable results. For the least of the three curvatures the ratio $\frac{R}{c}$ was 4 for which the effect of the curvature is small, and hence with this curvature the strength of the curved beam would be expected to be only slightly less than that of a straight beam having the same cross-section. Intermediate between these curvatures one was selected having a value of $\frac{R}{c}$ equal to 2. The cross-sections of all beams except the H section had as nearly as practicable the same sec-

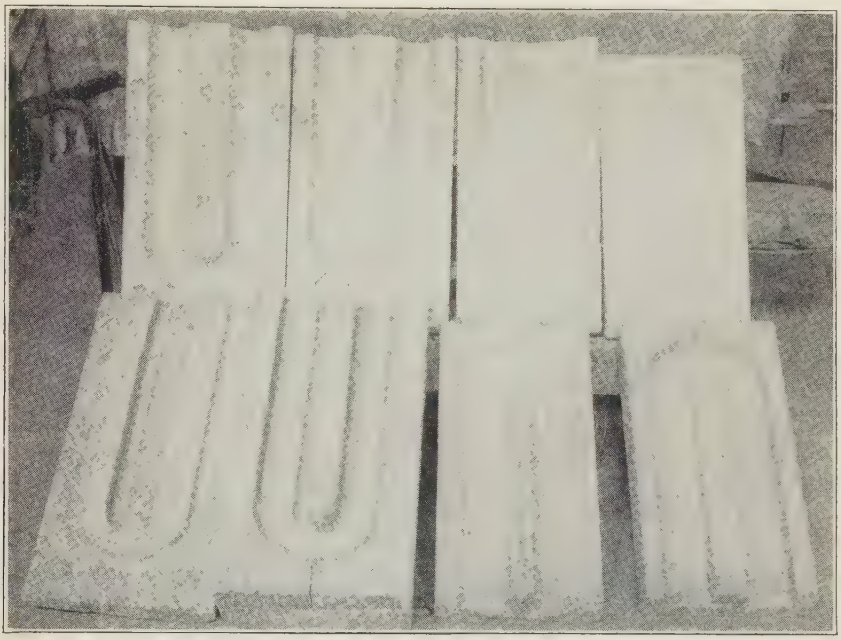


FIG. 4. MOLDS OPENED

tion modulus, $\frac{I}{c}$, namely, a value of 3 in.^{3} which for a circular cross-section requires a diameter of $3\frac{1}{8}$ in. The section modulus for the H section was 5.3 in.^{3}, approximately. The H section, which was made only with the sharpest of the three curvatures $\left(\frac{R}{c} = 1.6\right)$, was further proportioned so that the neutral axis (axis of zero stress) of the curved portion occurred at the junction of the flange and web (see Fig. 1), a condition requiring the computed radial tensile stress to be a maximum at the junction of flange and web. A rather high stress concentration also occurs at this abrupt change in width of section, and hence causes the most favorable condition for a radial tension failure in the web.

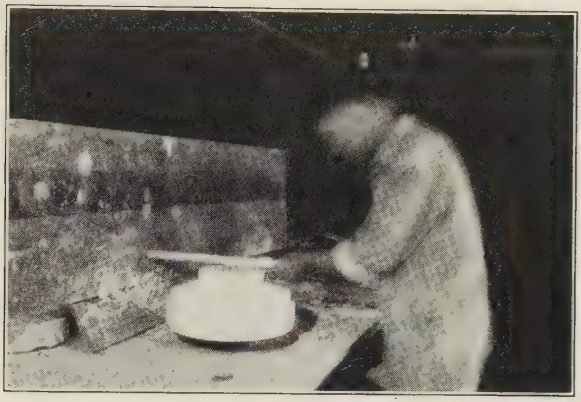
After a specimen was broken as a curved beam each of the straight portions was tested as a straight beam. In order to have a sufficient length to apply loads at the third points without causing too high shearing stresses, the straight portions were made approximately 16 in. long and were tested on a 15-in. span with one-third point loading, giving a constant moment in the middle third of the beam where failure occurred by bending.

7. *Molds for Casting Specimens.*—Molds were necessary for casting the specimens because a number of specimens were required for each section and for each degree of curvature, and their manufacture by machining from a rough block would have required too much time, although when feasible a machined specimen is to be desired; a view of some of the molds is shown in Figs. 3 and 4.

The technique of mold making improved with each set of molds made, and hence the same procedure was not followed exactly for any two sets.

In making the molds for the beams with the circular cross-section the pattern was made by pressing stiff clay through a circular die and then bending the resulting clay cylinder to the required shape. Before the clay was prepared the lay-out of the specimen was drawn with indelible pencil upon the marble top of the work table in the plaster shop. Plaster was poured over this drawing to a thickness of about one inch. When this had set sufficiently the resulting slab was turned over, and, on the face which had been next to the table there was a reproduction of the drawing on the table top. Using these lines to work to, a "parting" surface for the mold was cut from the slab. The next step was to prepare the clay. After some experimenting the proper consistency was found and cylinders were produced with a smooth surface and yet stiff enough to be easily handled. These were carefully bent to lie within the lines previously drawn upon the table top. A line was then drawn along an element of the bent cylinder at a height from the table equal to the radius of the cylinders. Then the plaster slabs that had been prepared for the parting surface were carefully placed in position with their upper surfaces in the plane of the line drawn on the clay cylinder. After the parting surface had been securely placed, the irregularities in the clay surface were corrected by means of a semicircular strike-off template, cut from a piece of sheet iron. The parting surface served as a guide for this operation.

When this clay form was completed the entire surface was "sized" thoroughly with "English soft soap," an outside form was placed around the set-up, and plaster poured in to form one-half of the mold. When the plaster had set sufficiently, the mold was turned over, the parting surfaces were removed, the clay cylinder projecting from the half mold was brought to proper size and finished as before. All surfaces were then thoroughly sized with the soap solution, the outside form was replaced, and the mold was completed by pouring plaster upon the upturned surface.



Turning Top Face

Turning Operation Completed. Ring Ready to be Sawed in Two

FIG. 5. METHOD OF FORMING PATTERN OF CURVED PORTION OF CURVED BEAM HAVING TRAPEZOIDAL CROSS-SECTION

The trapezoidal section and the H section presented new difficulties. The clay pattern was not a success for forming the curved portion of the beam. For these specimens the curved ends were turned on a potter's wheel in the following manner:

A strip of thin sheet iron was bent into a band with an inside diameter somewhat larger than the outside diameter of the curved portion of the proposed specimen. This band was centered upon a potter's wheel, which had been previously sized with soap, the crevices were chinked with clay, and plaster was poured into the band.

As soon as the plaster began to harden the band was removed, the wheel was placed upon the spindle, and the turning process was begun (see Fig. 5). A saucer with a cylindrical (slightly tapered) projection in the center, with a diameter equal to the inside diameter of the proposed beam, was first turned. This saucer was to form one face of the section. When the saucer and core were finished the surface was sized thoroughly with the soap solution, the band was replaced and more plaster was poured in to the desired depth. This time the outside and top of the curved portion was cut at the proper height above the edge of the saucer. When completed the turned ring was sawed in two on a diameter and one semi-circular portion formed the curved portion of the curved-beam pattern. The straight portions for the pattern were formed of stiff clay pressed through a die as had been done for the circular section. This mold was made in four parts.

The general procedure for turning the curved portion of the pattern for the beam of H section was the same as that for the trapezoidal section. The pattern for the straight portions was made by sawing and dressing small slabs of plaster to the dimensions of web and flanges. These were held in position while the first part of the mold was being poured by pressing them into chunks of the clay.

From the experience gained from the manufacture and the use of these molds it is believed that the patterns for the curved portions of all four sections could have been most quickly and accurately made by turning as described above.

It is also believed to be worth while to mix and cure the plaster used in the molds so as to get the maximum practicable strength, especially if a relatively large number of castings are expected to be cast from one mold. The somewhat rough handling necessary to free the castings from the molds and the repeated wetting incident to use make the life of the plain plaster mold rather short, and for this reason Portland Cement was used in some later tests, with good results, for making molds to withstand considerable usage.

III. Method of Testing

8. *Testing Machines Used.*—About one-half of the specimens broken were tested in a Scott horizontal testing machine of 1000-lb. capacity (Fig. 6), and the other half were tested in an Olsen vertical screw machine of 10 000-lb. capacity, using a poise for a maximum load of 1000 lb. (Fig. 12a). Both machines were satisfactory, and the change in machines was made merely as a matter of convenience.

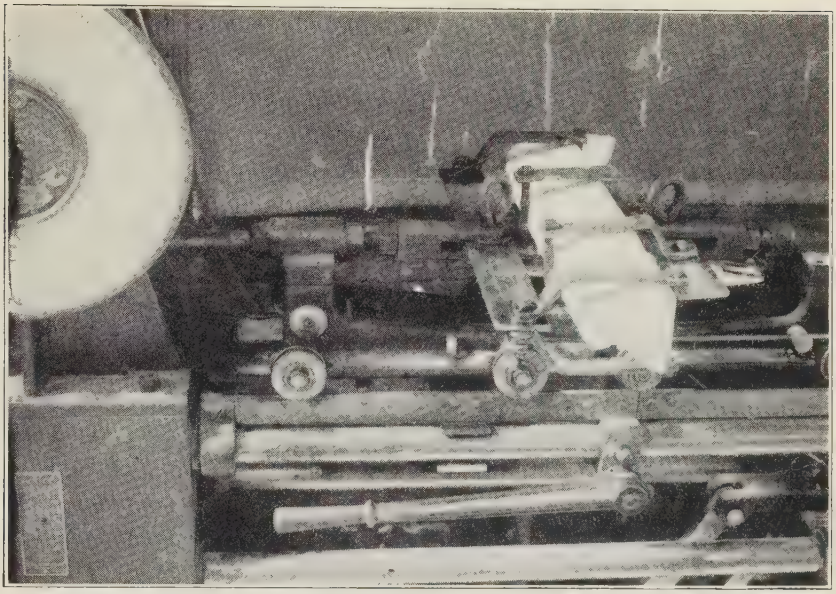


FIG. 6. METHOD OF TESTING STRAIGHT BEAMS

Both machines were calibrated carefully and were found to be satisfactory in accuracy and in sensitiveness.

9. *Method of Loading.*—The load applied to the curved beam specimens had a moment arm of 15 in. (see Figs. 2 and 12). If the moment arm were made large the specimen would break at a very low load which would be difficult to measure accurately, and if made small the stress in the curved portion of the beam would not be due mainly to bending, that is, the direct stress would be relatively large, which was considered to be objectionable. A moment arm of 15 in. was considered to be a satisfactory compromise for these conflicting conditions. The average of the loads causing failure of the curved-beam specimens was about 50 lb. and that causing failure of the straight beams was about 600 lb.

Each curved-beam specimen after being broken as a curved beam furnished two straight-beam specimens. The straight-beam specimens were tested as simple beams with a span of 15 in. The loads were applied at the third points, thus giving a 5-in. length in the center of the beam that was subjected to a constant bending moment. The failure of nearly all of these straight beams occurred in the middle

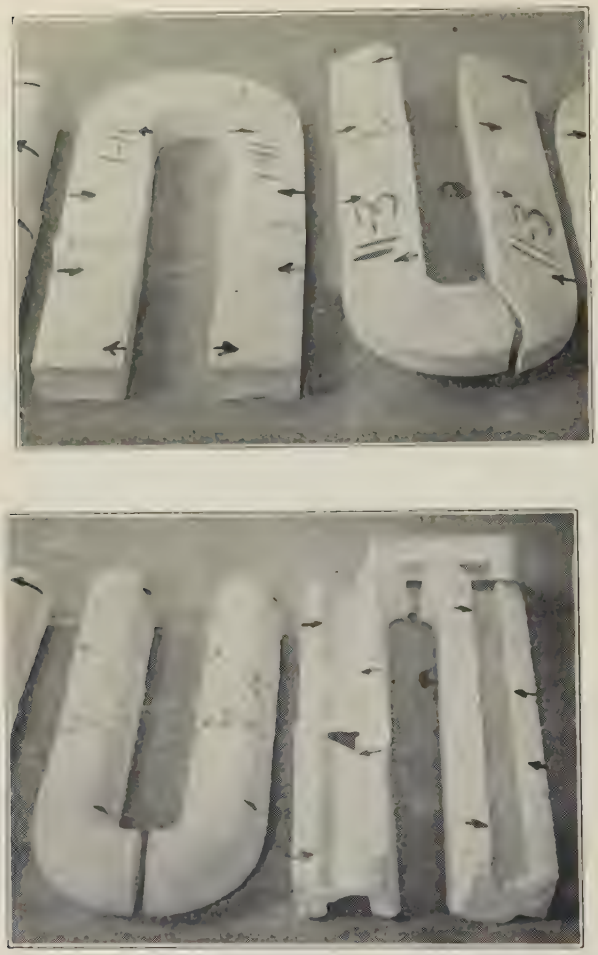


FIG. 7. SOME OF THE SPECIMENS AFTER TESTING

third of the beam and was due to bending; failure due to shear or to diagonal tension in the outer thirds seldom occurred. Figure 6 shows the method of loading the straight beams and Fig. 12a the method of loading the curved beams. Some of the broken specimens are shown in Fig. 7.

IV. RESULTS AND DISCUSSION

10. *Experimental Values of Correction Factor.*—It was stated in Section 2 that one of the main objects of the investigation was to deter-

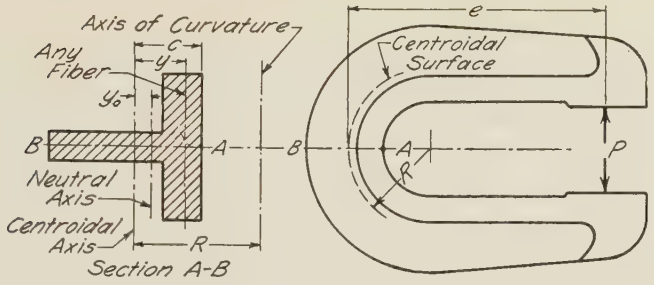


FIG. 8. BRIEF EXPLANATION OF WINKLER-BACH FORMULA

mine whether or not the plaster-model method would yield consistent and reliable values for the maximum stress in curved beams.

This object may be accomplished by comparing the theoretical or computed value of the so-called "correction factor" for a curved beam with the experimental value. The theoretical correction factor for a curved beam is the number by which the value of s in the formula $s = \frac{Mc}{I}$, which is applicable to straight beams only, must be multiplied to give the stress in a curved beam having the same cross-section and resisting the same bending moment.

The value of the maximum stress in a curved beam is considered to be given satisfactorily by the Winkler-Bach formula:

$$s_c = \frac{P}{a} + \frac{M}{ar}\left(1 + \frac{1}{Z}\,\frac{c}{R+c}\right) \quad \cdots\cdots \quad (1)$$

The derivation of this formula and the meaning of the terms are discussed in Circular 16 of the Engineering Experiment Station of the University of Illinois. Figure 8 gives a brief explanation of the formula.

The value of the theoretical correction factor k then is the ratio of s_c to s in the preceding formulas. Values of the theoretical correction factors for curved beams of various degrees of curvature and of various shapes of cross-section are given in Circular 16, and these values have been used for comparison with the experimental values found from the tests herein reported.

The experimental values of the correction factor are found as follows: Let M_c denote the test value of the bending moment causing rupture of one of the plaster curved beams, and let M_s denote the test value of the bending moment that causes rupture of one of the two straight beams obtained from the same curved-beam specimen. If now M_s is k times as large as M_c, then for any given bending moment the maximum stress in the curved beam is k times as large as that in the straight beam since the straight beam and the curved beam supposedly break when the maximum stress in each has reached the same value, namely, the ultimate strength of the material. The experimental value of the correction factor k then is the ratio $\frac{M_s}{M_c}$. The breaking stress in the curved-beam specimens, however, is not due solely to bending, for there is a small direct stress $\frac{P}{a}$ equal to the breaking load divided by the area of the section. Therefore, the value of M_c used in obtaining values of k is found by adding to the value of M_c corresponding to the breaking load a moment sufficient to cause, at the most stressed fiber, a stress equal to the direct stress. The total moment is denoted by M_c' and differs but little from M_c, for the moment added to correct for the direct stress is small in all cases. The experimental correction factor then is

$$k = \frac{M_s}{M_c'} \quad \cdots\cdots\cdots \quad (2)$$

and values of the correction factor for the various curved beams tested are given in Table 1.

In Table 2 is given the average of the experimental values of the correction factor for each of the several curved-beam sections used, together with the corresponding theoretical correction factors taken from Circular 16.

The experimental values of the correction factor in Tables 1 and 2 show that there is a rather large variation in the values for the specimens in any group but that the average values of the correction factor are consistent with the theoretical values.

The results in Table 2 justify the conclusion that the plaster-model method may be used with confidence in obtaining an approximate but useful value of the maximum unit stress even in a member of rather complicated form, provided that the average of the results of tests of a relatively large number of pairs of specimens is used.

TABLE 1

RESULTS OF TESTS OF BEAMS THAT FAILED BY CIRCUMFERENTIAL STRESS

(All quantities expressed in pound and inch units)

No.	M_c'	s_c	M_s	s	$k = \frac{M_s}{M_c'}$
Rectangular Section, $\frac{R}{c} = 1.6$					
1-1	1011	588	1415	460	1.40
			1537	499	1.52
5-1	1302	757	1850	602	1.42
			1982	643	1.52
7-1	1026	597	1727	561	1.68
			2080	675	2.03
10-1	812	473	1565	500	1.93
			2002	650	2.47
13-1	1226	714	1612	524	1.31
			1895	614	1.54
14-1	1041	606	1712	556	1.64
			1830	594	1.76
15-1	1409	820	1532	498	1.09
			1700	553	1.21
17-1	982	577	1300	414	1.32
			1215	387	1.24
19-1	811	483	1475	490	1.82
			1614	536	1.99
20-1	1072	618	1593	504	1.48
			1457	461	1.36
21-1	796	459	1637	518	2.06
			1719	544	2.16
23-1	796	459	1637	518	2.06
			1612	510	2.03
29-1	810	487	1589	506	1.96
			1520	484	1.88
Average:		586		531	1.70
Circular Section, $\frac{R}{c} = 1.6$					
1-2	699	459	1837	606	2.62
			1562	516	2.23
5-2	1140	749	2020	660	1.77
			1737	568	1.68
7-2	1277	838	1957	640	1.52
			1667	545	1.31
10-2	1155	758	2005	650	1.74
			1792	580	1.55
Average:		701		596	1.80
H Section, ½-in. Fillet, $\frac{R}{c} = 1.6$					
44-7	1052	368	2749	497	2.62
			2092	378	1.98
45-7	898	314	2179	394	2.42
			2175	393	2.42
46-7	851	298	2617	473	3.08
			2385	431	2.81
Average:		327		428	2.55

No.	M_c'	s_c	M_s	s	$k = \frac{M_s}{M_c'}$
Rectangular Section, $\frac{R}{c} = 2.0$					
2-1	952	483	1432	460	1.51
			1575	506	1.65
4-1	893	454	1465	471	1.64
			1350	434	1.51
6-1	1505	764	1995	639	1.33
			1717	550	1.14
11-1	1029	522	2170	697	2.11
			2075	666	2.02
....	1236	627	1307	420	1.07
			1695	544	1.37
18-2	1011	508	1253	416	1.24
			1418	471	1.40
19-2	753	362	1481	484	1.97
			1515	495	2.01
20-2	829	424	1441	491	1.74
			1256	428	1.51
21-2	1090	555	1398	476	1.28
			1460	497	1.34
22-2	845	426	1060	352	1.25
			1626	540	1.92
23-2	814	410	1511	502	1.86
			1385	460	1.70
Average:		504		500	1.57
Circular Section, $\frac{R}{c} = 2.0$					
2-2	1104	587	1700	570	1.54
			1830	614	1.66
4-2	1151	612	2087	700	1.81
			1865	625	1.61
6-2	1388	738	1950	667	1.40
			2112	722	1.52
9-2	1097	583	1790	612	1.63
			1857	635	1.69
11-2	1097	583	2012	658	2.01
			1995	652	1.82
Average:		620		646	1.67
Trapezoidal Section, $\frac{R}{c} = 2.0$					
2-3	1028	503	1700	543	1.65
			2050	621	1.64
4-3	1250	612	2295	665	2.02
6-3	1135	556	1805	534	1.59
9-3	1427	698	2227	659	1.56
11-3	1028	503	1795	520	1.74
Average:		575		590	1.70

TABLE 1.—(CONCLUDED)

RESULTS OF TESTS OF BEAMS THAT FAILED BY CIRCUMFERENTIAL STRESS

(All quantities expressed in pound and inch units)

No.	M_c'	s_c	M_s	s	$k = \frac{M_s}{M_c'}$
Rectangular Section, $\frac{R}{c} = 4.0$					
3-1	1249	420	1482	494	1.19
			1757	586	1.41
12-1	1443	485	2195	703	1.52
			1875	600	1.30
0-3	853	373	1244	431	1.46
			1264	438	1.48
17-3	993	421	1564	498	1.58
			1244	396	1.25
18-3	987	419	1219	388	1.23
			....	...	....
19-3	1022	382	1227	409	1.20
			1224	408	1.20
20-3	774	289	1149	383	1.44
			1443	481	1.84
21-3	836	315	1035	345	1.24
			1056	352	1.26
22-3	867	326	1080	360	1.25
			1008	336	1.16
23-3	759	282	1170	390	1.54
			1209	403	1.59
Average:		371		442	1.37

No.	M_c'	s_c	M_s	s	$k = \frac{M_s}{M_c'}$
Circular Section, $\frac{R}{c} = 4.0$					
3-2	....	...	2290	741	....
8-2	1668	485	....	...	....
12-2	1575	458	2140	692	1.36
			2287	740	1.45
Average:		476		724	1.40
Trapezoidal Section, $\frac{R}{c} = 4.0$					
3-3	1462	521	2025	614	1.38
			1812	549	1.24
8-3	1510	538	1825	553	1.21
			1725	525	1.14
12-3	1446	515	....	....	....
			1710	505	1.18
Average:		525		550	1.23

It is equally clear that the results of tests of a single pair of specimens may be very misleading at least with members of rather complicated form.

The requirement of a relatively large number of tests is not a serious objection to the plaster-model method for the reasons that a large number of specimens is easily obtained after the molds are made, and the testing of the specimens is done quickly with very little labor and equipment.

11. *Conditions Affecting Results.*—To one accustomed to the uniformity of results obtained from tests on ductile steel, the variations in results obtained from tests of pottery plaster may seem to indicate that the plaster-model method of determining the maximum stress in a member can hardly be expected to yield reliable results.

In judging of the significance of the statistical value obtained by averaging the results of a number of tests of plaster models it is well to observe that the results from a single test of ductile steel may be, to a considerable extent, a statistical value; for the relatively large yielding and internal adjustments in the ductile steel at probably many points of high stress concentrations cause, by this mechanical process, the result from the single test to be an average or statistical

TABLE 2

AVERAGE EXPERIMENTAL VALUES AND THEORETICAL VALUES OF CORRECTION FACTOR

Section	$\frac{R}{c}$	Theoretical Value	Average Experimental Value	Max. and Min. Values		Num. of Values Averaged
				Max.	Min.	
Rectangular	1.6	1.79	1.70	2.47	1.09	26
Circular	1.6	1.96	1.80	2.62	1.31	8
H Section (1/8-in. fillet)	1.6	1.93	2.55	3.08	1.98	6
Rectangular	2	1.52	1.57	2.11	1.07	22
Circular	2	1.62	1.67	2.01	1.40	10
Trapezoidal	2	1.60	1.70	2.02	1.56	6
Rectangular	4	1.20	1.37	1.84	1.16	19
Circular	4	1.23	1.40	1.45	1.36	2
Trapezoidal	4	1.25	1.23	1.38	1.14	5

value. In a brittle material like pottery plaster, however, in which the fortuitous stress concentrations due to variability of material are greater and can be relieved to a much less extent, these stress concentrations play a larger part in determining the strength of the specimen than in a ductile metal. But it is to be expected that an average of the results of a number of tests would be consistent and reliable, as is indicated by the test results herein recorded.

The reliability of the results of the plaster-model method depends mainly on having the material in the model of complex shape in which the maximum stress is to be found (the curved portion of the C-shaped specimen) as nearly as possible like that in the specimen of simple shape (the straight portions of the C frame) in which the maximum stress can be calculated.

In order to secure this condition, care must be exercised in stirring and pouring the mixture to the end that all large air bubbles will be removed, and the minute air spaces or voids will be evenly distributed throughout the specimen. The method of curing and of drying are also of great importance in this respect. With the C-shaped specimens used in these experiments some difficulty was experienced in drying uniformly, due partly to the large size and relatively complex form of the specimens.

If the material in a curved beam specimen is the same throughout and the material has a straight stress-strain diagram up to the ultimate stress, then the stress at which the curved beam breaks (modulus of rupture) should be the same as that at which each of the two straight-beam specimens breaks.

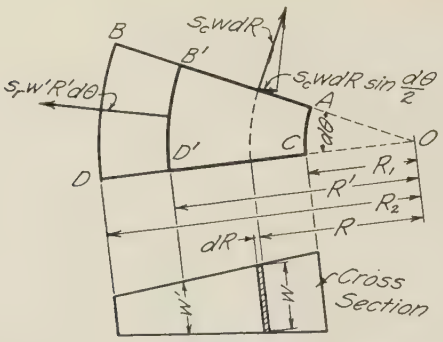


Fig. 9. Radial Tensile Stress in Curved Beams

From an inspection of the values of the modulus of rupture given in Table 1, however, it will be seen that for some of the specimens the modulus of rupture for a curved beam differs considerably from that obtained from either of the two straight portions of the same specimen; and also it will be seen that the breaking stresses for the two straight beams from a given specimen in some cases vary considerably.

If the plaster-model method were used with specimens of simpler shape, requiring simpler molds, or if the specimens were turned from a block of simple shape, the variation in the material in each pair of specimens would probably be less than that found in these tests due to greater ease in avoiding air bubbles and in curing and drying evenly, and more reliance could probably then be placed on the results of the tests of a single pair of specimens.

12. *Transverse or Radial Stress.*—The second purpose of the investigation, as stated in Section 2, was to investigate the radial stress in a curved beam having a section with an abrupt change in width such as a T section, an I section, or an H section. For such sections the mathematical or theoretical value of the radial tensile stress may be larger than the circumferential stress, and also with such sections the concentration of radial stress may be large.

Theoretical Radial Stress

A mathematical expression for the radial stress at any point in the cross-section of a curved beam as suggested by Winslow and Edmonds* may be obtained briefly as follows: In Fig. 9 let *ABDC* rep-

*See Bibliography. For a more elaborate analysis see Case, "Strength of Materials," 1925, p. 431. Longmans Green Co.

resent a small portion of a curved beam near to the section AB in Fig. 8. The two normal sections AB and CD make an angle $d\theta$, and on these sections only the normal stresses due to bending are considered; the shearing stress is considered to be negligible. Let the normal bending unit stress at any point in the section at the distance R from the center of curvature O be s_c. The total stress on an element of area $w\ dR$ then would be $s_c\ w\ dR$. Further, let the radial (tensile) unit stress on a circumferential section at the distance R' from O be s_r; then the total stress on this section is $s_r\ w'\ R'\ d\theta$.

By considering the equilibrium of the portion $AB'D'C$ we may write

$$s_r w' R' d\theta = 2 \int_{R_1}^{R'} s_c w dR \sin \frac{d\theta}{2} \quad \cdots\cdots \quad (3)$$

But for a small angle $\sin \frac{d\theta}{2} = \frac{d\theta}{2}$

whence,

$$s_r = \frac{1}{w'R'} \int_{R_1}^{R'} s_c w dR \quad \cdots\cdots \quad (4)$$

The value of s_r is zero at the inner surface because R' becomes equal to R_1 and hence the integral in Equation (4) is equal to zero. Likewise, s_r is zero at the outer surface since the total radial stress on the whole cross-section, and hence the integral in Equation (3), is equal to zero because there are no radial external forces. The manner in which s_r varies between the inner and outer surfaces depends on the variation in the width w; when w is small s_r is large. The value of the summation represented by the integral in Equation (4) for any distance R' may be found by dividing the cross-section into narrow strips of known area wdR and multiplying the area of each strip by the value of the circumferential unit stress s_c at the strip, and adding all such products for the strips between R_1 and R'.

The values of the theoretical radial and circumferential stresses for a number of sections are shown in Figs. 10 and 11, and it will be noted that the theoretical radial stress at abrupt changes in width of a section may be greater than the maximum theoretical circumferential stress as given by the Winkler-Bach formula. When in addition to this condition the concentration of stress at the abrupt change in width of the section is considered the importance of the radial stress becomes evident.

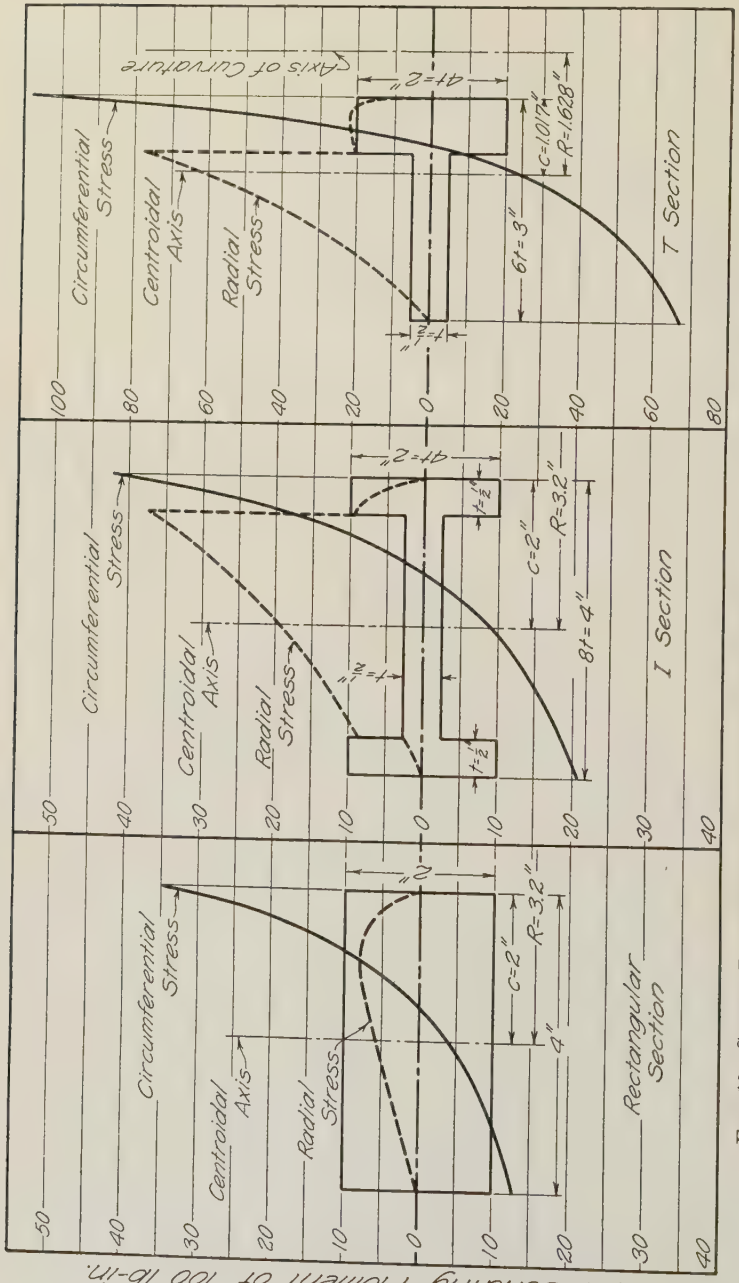


FIG. 10. STRESS DISTRIBUTION CURVES FOR RECTANGULAR, T, AND I SECTIONS, FOR $R/c = 1.6$

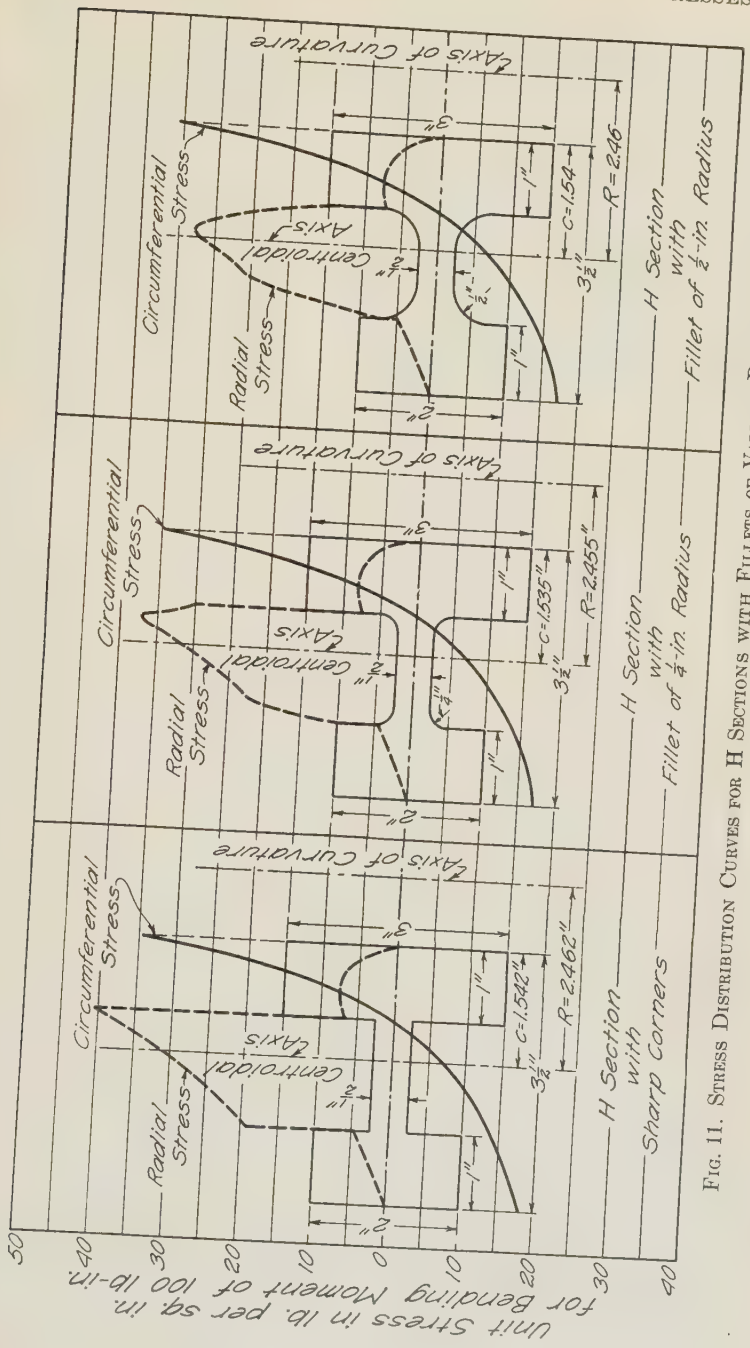
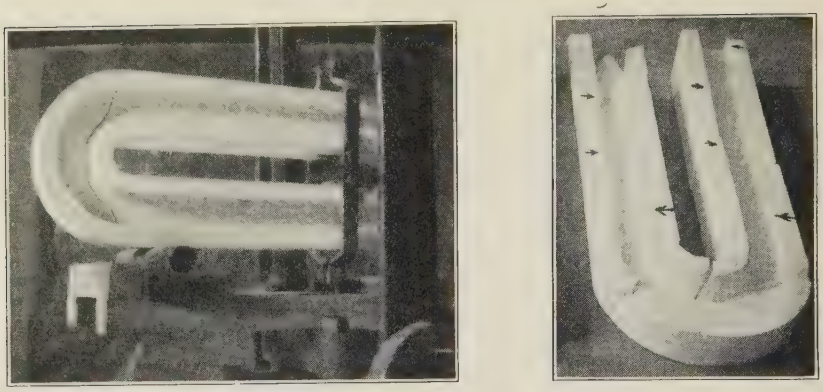


FIG. 11. STRESS DISTRIBUTION CURVES FOR H SECTIONS WITH FILLETS OF VARIOUS RADII FOR $R/c = 1.6$



Initial Failure Final Failure

FIG. 12. SPECIMENS SHOWING RADIAL STRESS FAILURE

Experimental Values of Radial Stress

As noted in Section 2, a curved beam with an H section was used to determine the effect of the radial stress. This H section is shown in Fig. 1.

One group of eight specimens was made with an H section having a sharp corner at the abrupt change in width; another group of three specimens had a fillet of $\frac{1}{8}$-in. radius instead of a sharp corner; another group contained five specimens having sections with a fillet of $\frac{1}{4}$-in. radius; and another group was composed of three specimens having sections with a fillet of $\frac{1}{2}$-in. radius. These sections, with the exception of the one with the $\frac{1}{8}$-in. radius, are shown in Fig. 11. All of these specimens failed in radial tension except the three having a fillet of $\frac{1}{2}$-in. radius; these three failed by circumferential tension.

The failure of a specimen by radial tension always occurred at the junction of the flange and web, and was accompanied by a rather sharp cracking noise, and immediately after this a fine-lined crack could be observed extending a short distance along the juncture of flange and web and gradually extending, as the machine continued to run, until it reached completely across the web as shown by the dark line in Fig. 12a. No additional load was carried by the specimen after the first crack occurred. The ultimate failure of the specimen took place by the rupture of the flanges (see Fig. 12b).

The values of the radial unit stress s_r corresponding to the breaking load as obtained from Equation (3) are given in Table 3. It will be noted that these values are much less than the breaking stress of the material as found from the tests of the straight beams. This fact

TABLE 3

RESULTS OF TESTS OF H BEAMS THAT FAILED BY RADIAL TENSION

$\left(\frac{R}{c} = 1.6 \text{ for all the beams; values expressed in lb. and in. units}\right)$

No.	M_c'	s_c'	M_s	s	s_r	$q = \frac{s}{s_r}$
Radius of fillet = 0, (sharp corner, Fig. 11)						
13-4	589	213	2990	570	234	2.44
14-4	775	279	....	...	....	....
15-4	961	347	2411	453	307	1.48
			2857	537	382	1.41
			2193	418		1.10
16-4	620	221	2983	560	246	2.28
			2730	521		2.12
24-4	449	165	1280	252	182	1.38
			1976	389		2.14
28-4	663	230	2218	413	242	1.71
			2529	471		1.94
29-4	760	264	2395	446	263	1.70
			1783	332		1.27
31-4	527	183	2323	440	190	2.32
			2381	451		2.38
39-4	519	177	2301	435	191	2.28
			2280	431		2.26
42-4	279	95	1399	265	108	2.46
			871	165		1.53
33-4	403	132	1991	377	190	1.98
			1837	348		1.83
Average:				413	230	1.90
Radius of fillet = ⅛ in.						
30-5	744	256	1200	226	257	0.88
			2103	396		1.54
33-5	931	306	1500	293	310	0.95
			1787	349		1.13
34-5	652	208	1908	360	199	1.83
			1924	363		1.84
Average:				332	283	1.36
Radius of fillet = ¼ in. (see Fig. 11)						
39-6	1106	400	2312	442	372	1.18
			2338	447		1.20
42-6	789	285	2066	395	265	1.49
33-6	867	299	1893	362	282	1.28
			1935	370		1.31
34-6	960	337	1700	319	312	1.02
			1737	326		1.04
44-6	1037	375	2594	497	369	1.34
			2464	472		1.28
Average:				407	320	1.24

indicates that there is considerable stress concentration at the abrupt change in width of section at the juncture of web and flange.

13. *Stress Concentration Factors.*—The factor by which the theoretical radial stress corresponding to the breaking load must be multiplied to give the value of the breaking stress or modulus of rupture of the material as obtained from the tests of the straight beams with the

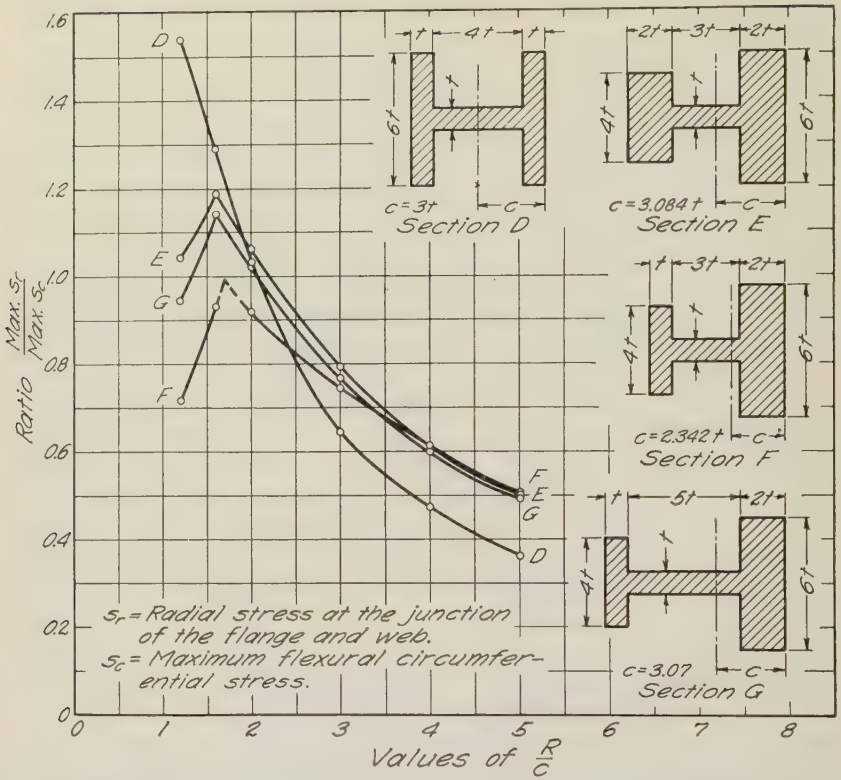


Fig. 13. Effect of Curvature of Beam on Ratio of Maximum Radial Stress to Maximum Circumferential Stress for Each of Several H Sections

same cross-section is called the radial stress concentration factor. The radial stress distribution across the web at the juncture of web and flange instead of being uniformly distributed as is assumed in deriving the theoretical formula is higher than the theoretical value at the edges for all the sections except those having the one-half inch fillet; with this large fillet the stress concentration is evidently negligible since none of these specimens failed by radial tension, although the theoretical radial stress at the junction of web and flange, as shown in Fig. 15, is nearly equal to the maximum circumferential stress.

In Table 3 are given the values of the stress concentration factors for all the specimens having H sections, and the average for each group of specimens having the same section is given in Table 4. The number of tests for the sections with fillets is not sufficient perhaps to

TABLE 4

AVERAGE EXPERIMENTAL VALUES OF RADIAL STRESS CONCENTRATION FACTORS FOR H SECTIONS WITH FILLETS

Radius of Fillet ρ	Ratio $\frac{\rho}{d}$ (d = width of web)	Number of Tests Averaged	Factor of Radial Stress Concentration $q = \frac{s}{s_r}$	Values of Factors of Longitudinal Stress Concentration for Fillets in Straight Beams as Found by:	
				Plaster-model Method†	Polarized Light Method‡
0 (sharp corner)	..	11	1.90	1.88	....
1/8	1/4	3	1.36	1.27	1.73
1/4	1/2	5	1.24	1.23	1.40
1/2	1	3*	1.00*	....	....

*All three specimens failed by circumferential stress (not by radial stress) and the calculated value of the radial stress for each beam was nearly equal to the breaking stress for the material as found from the tests of the straight beams from the same specimens; therefore there was little if any concentration of stress and consequently the factor of stress concentration is given as unity.

†Peterson, R. E. (see Bibliography)

‡Timoshenko and Dietz. Proc. A.S.M.E. May, 1925.

obtain an average value that can be considered entirely reliable but the values are very nearly the same as those found by Peterson* by the plaster-model method for similar fillets in straight beams, and are consistent with the values found by Timoshenko and Dietz* by the polarized light method for similar fillets. The values obtained by the polarized light method are always found to be larger than those obtained by a method in which the slight adjustment or yielding of the member or model influences the results.

It seems clear from the foregoing results that the radial stress cannot always be neglected in the design of curved beams. For beams made of relatively brittle material like cast iron and having sections with abrupt changes in width, the radial stress, rather than the circumferential stress, may be the significant stress. If the beam is subjected to a steady or static load and is made of ductile material, such as low carbon steel, the concentration of radial stress at the abrupt change in width of section may be considerably relieved by the local yielding without seriously damaging the member as a whole; but if the load is repeated many times the concentration of stress is of great importance even with very ductile material, for a crack is likely to start at the point of high stress concentration and to spread gradually across the area until failure occurs. In any case the section should be designed, if possible, to prevent a high value of the radial stress.

*See the Bibliography.

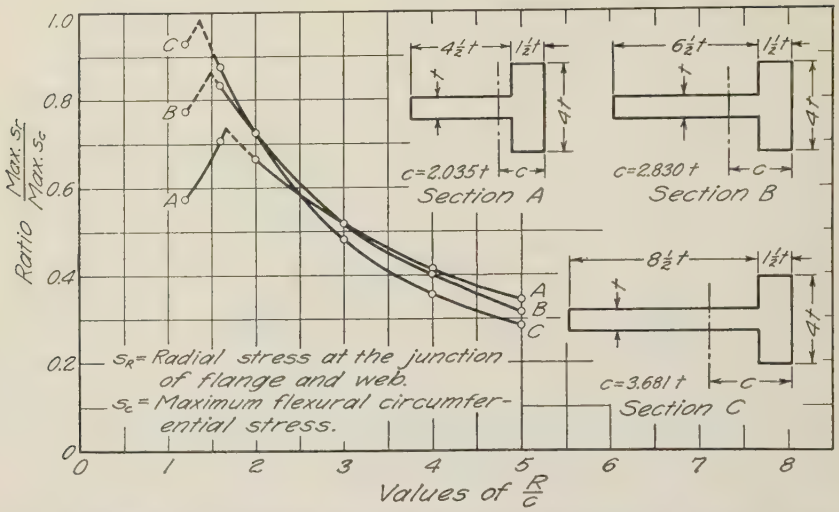


Fig. 14. Effect of Curvature of Beam on Ratio of Maximum Radial Stress to Maximum Circumferential Stress for Each of Several T Sections

14. *Estimated Value of Maximum Radial Stress In Any Section.*— Since the computation for the radial stress in a curved beam is rather tedious, it is desirable to be able to estimate, without computation, the value of the maximum radial stress in curved beams of different sections having abrupt changes in width, or at least to be able to determine approximately whether or not the radial stress may be the significant stress.

For this purpose Figs. 10 to 11 and Figs. 13 to 15 may be helpful, when used in connection with Table 4 giving the stress concentration factors for fillets. Figure 13 shows, for each of several H sections, how the maximum radial stress varies with the curvature of the beam. Figure 14 gives the same information for each of several T sections. Figures 13 and 14 also show in a general way how the maximum radial stress in an H section and in a T section varies with the dimensions of the section for a curved beam with any given curvature. Figure 15, however, shows quantitatively the effect on the maximum radial stress in relation to the maximum circumferential stress in an H section and in a T section of varying the web thickness, but maintaining a constant value of $\frac{R}{c}$ of 1.6. It must be remembered that the values of $\frac{s_r}{s_c}$ given in these figures must be increased according to the values of

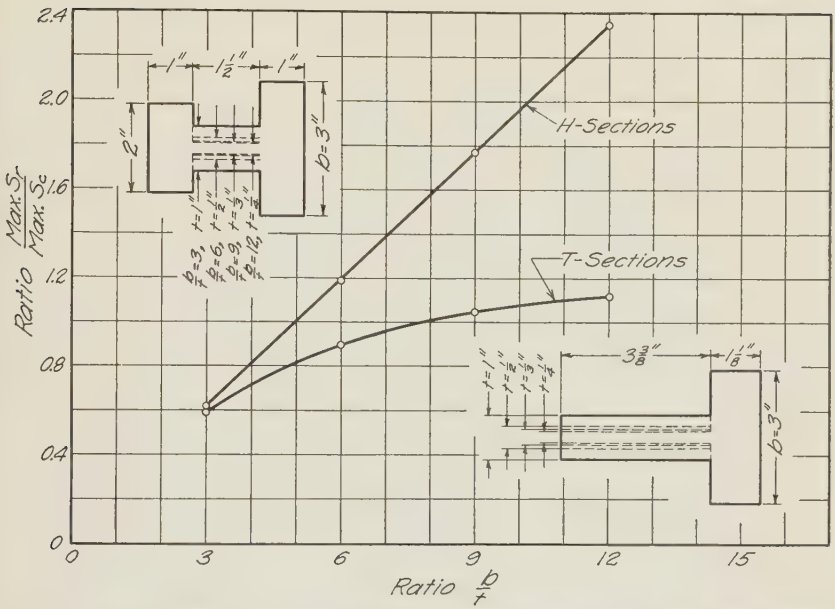


Fig. 15. Effect on Ratio of Maximum Radial Stress to Maximum Circumferential Stress in an H Section and a T Section of Changing Web Thickness of the Section, for a Beam with $R/c = 1.6$

the stress concentration factors given in Table 4 if the material is rather brittle or even if the material is ductile and is subjected to repeated loads. If, however, the material is ductile and is subjected to static loads, the effect of the stress concentration at the abrupt change in width of the section is of much less importance.

Figures 13, 14, and 15 indicate that the radial stress in a curved beam having a section composed of flanges and web will probably be of less importance than the circumferential stress for all values of $\frac{R}{c}$ greater than about 2, provided that there are liberal fillets at the abrupt change in width of section so that there is little or no stress concentration, and provided also that the thickness t of the web is not less than about one-fifth the width b of the flange.

V. Conclusion

15. *Summary.*—A summary of the main facts concerning the investigation may be given as follows:

(1) Pottery plaster, if properly cured and dried, has approximately a straight stress-strain diagram up to the ultimate or breaking stress; its behavior up to rupture, therefore, is approximately the same as the elastic behavior of a ductile metal, like steel.

(2) The pottery-plaster method of determining the maximum stress in a member caused by given loads may be stated very briefly as follows: A pair of specimens of pottery plaster are made, one specimen being a model of the member in which the stress is to be found, and the other a model of simple form for which the relation between load and stress is known. The two specimens are tested to rupture. From the known relation of load and stress in the specimen of simple shape and the ratio of the loads (or bending moments) causing rupture of the two specimens the relation of maximum stress and loads in the model of complex shape is found. The same relation is assumed to hold in a similarly shaped member made of ductile material provided that the stresses are within the proportional limit of the material.

(3) It is feasible to make pottery-plaster specimens of rather complicated forms in plaster or concrete molds from which a number of specimens may easily be made.

(4) The pottery-plaster method gave consistent and reliable values for the maximum stress in curved beams of the various sections, provided the average value of the results of a number of tests was used. The results of the tests of a single pair of specimens may be misleading.

(5) The theoretical transverse or radial tensile stress in a curved beam having a cross-section in which there is an abrupt change in width may be larger than the circumferential tensile stress.

(6) The concentration of radial tensile stress at the abrupt change of width of section of the curved beam was found to vary with the radius of fillet about as indicated in tests of other types of members where similar fillets have been used. All the curved beams that had an abrupt change in width of section (H sections) broke in radial tension at the abrupt change in width of section, except those having a radius of fillet equal to $\frac{1}{2}$ in. With this large radius of fillet, the stress concentration evidently became negligible.

(7) There is justification for the conclusion that an approximate but useful and reliable value of the maximum stress (within the pro-

portional limit) in a machine or structural member of relatively complicated form, made of elastic material, may be found by use of the plaster-model method provided that the average of the results of tests of a number of pairs of specimens is obtained. The number of tests needed will, to some extent, depend on the form of the model in which the stress is to be found and on the method of forming the model, and especially on the method of curing and drying the material. The number of tests made should also be governed somewhat by the variation in the results obtained. It is likely, however, that from five to ten tests will, in general, be ample to yield reliable results.

(8) The necessity of a relatively large number of specimens and tests is not a serious objection to the plaster-model method of determining the maximum stress in a member because many specimens may be made from one mold, and the testing of the specimens is done quickly and with little expense.

APPENDIX

Bibliography

No.	Year	Author	Title and Reference
1	1919	Slater, W. A.	Committee Report, Proc. A.S.T.M., Vol. XIX, Pt. I, p. 349.
2	1925	Timoshenko and Dietz	"Stress Concentration Produced by Holes and Fillets," Proc. A.S.M.E., May, 1925.
3	1925	U. S. Gypsum Co.	"Fundamental Factors in the Use of Pottery Plaster," Ceramic Industry, March, pp. 216-222.
4	1926	Peterson, R. E.	"An Investigation of Stress Concentration by Means of Plaster of Paris Specimens," M.S. Thesis, Univ. of Ill; also Mech. Eng., December, 1928, p. 1449.
5	1927	McVay, T. N. Breen, J. P.	"Highest Temperature at Which to Dry Plaster Models," Thesis, Univ. of Ill.; also Ceramic Industry, March, 1925, pp. 216-222.
6	1927	Wilson, B. J. Quereau, J. F.	"A Simple Method of Determining Stress in Curved Flexural Members," Univ. of Ill. Eng. Exp. Sta. Cir. 16.
7	1927	Winslow, A. M. Edmonds, R.H.G.	"Tests and Theory of Curved Beams," Mech. Eng., Vol. 49, p. 1097; also Bul. 46, Eng. Exp. Sta., Univ. of Wash.
8	1928	Rathbun, J. Charles	"Crown Stresses in a Skew Arch," (Plaster of Paris models used to test the reliability of the mathematical theory) Proc. A.S.C.E., Feb.

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